Only One Fitts' Law Formula – Please!

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CHI 2010, April 10–15, 2010, Atlanta, Georgia, USA.
ACM 978-1-60558-930-5/10/04.

Abstract
The HCI community uses at least four different formulas for Fitts' law. Each of them is derived from Shannon's information theory. This raises the question which formula is wrong and which is right. While the HCI community on the one hand gives free choice for the formula, it demands good statistical values for the evaluation on the other hand. From a scientific point of view this situation is not satisfying.

Keywords
Fitts' law, correlation.

ACM Classification Keywords
H5.2. Information interfaces and presentation (e.g., HCI): User Interfaces (Evaluation/Methodology).

General Terms
Theory

Introduction
Importance of Fitts' Law
Fitts' law gives a relation for the mean time to point at a target of given size in a given distance. Pointing is a very frequent action when interacting with computers using a mouse device. For this reason Fitts' law is very important in the field of HCI (Human Computer Interaction).

Another reason why Fitts' law is so popular in the HCI community seems to lie in the fact that it sees itself as a scientific community. Fitts' law has its foundation in information theory and therefore it relates to hard science. It is one of the few human-centric interaction formulas. As many aspects of HCI deal with the art of design and soft factors due to human nature, Fitts' law serves to support the scientific claim.

There are several hundred publications on Fitts' law reflecting this high importance. MacKenzie published a "Bibliography of Fitts' Law Research" on the internet with 310 entries (last updated in 2002). With this high number of publications it is difficult to keep the overview. It seems that the big amount of research papers contribute more to confusion than to a clarification of the topic. In particular, some publications introduced alternative formulas for Fitts' law.

It would be a benefit for the community if it could agree to one formula and if this high number of publications could be condensed to few relevant ones or to a chapter in a standard textbook.

Successful Fitts' Law Submissions

Whenever somebody submits a paper on an interaction technique which involves pointing, the reviewers like to see a Fitts' law evaluation. The recipe for a successful submission is:

Do evaluations with all formulas and for the submission chose the one which produces the best correlation. The correlation does not tell much, but the reviewers like to see a good correlation, e.g. close to 1. The easiest way to achieve a good correlation is the use of a small number of IDs. Two IDs only would produce a perfect correlation of value 1. With three IDs the good correlation value is still too obvious, but four or five IDs are fine. Definitively do not use more than seven IDs because this will lead to a poor correlation.

This sounds rather cynical than scientific but seems to be common practice. The situation represents a dilemma to the scientist: play the game to get a publication or stay at the confession and be never published. It makes it also difficult for the teacher, who should not tell this recipe to the students.

Of course, there is no hint that authors of publications took the formula with the best correlation, but it is sure that different publications use different formulas. A good reason, however, for the choice of the formula is missing in many cases.

Just as one example for argumentation with good correlations see the publication of Ashmore et al. [1].

"*Ware and Mikaelian [27] found that eye input does indeed follow Fitts' Law, although Zhai et al. [29] reported only low correlation to the model (r2 = .75). More recently, Miniotas [19] reported fairly high correlation (r2 = .98) with the following Fitts' variation, expressing mean selection time (MT) as: MT =a+blog2 (A/W +0.5), with a = 298 (ms) and b = 176 (ms/bit).*" [1]

The following sections report on Fitts' work, present a simple model to derive the formula, discuss variations of the formula and evaluation of data. After a discussion of the meaning of Fitts' law for HCI, the last section concludes that the HCI community should think about their scientific standards.

Fitts' Original Work

The research question of Fitts when he published his paper in 1954 was: "What is the limiting factor for the speed of aimed movements – the physical strength of the muscles or the information processing capacity of the nervous system?" Fitts observed that the performance time did not change much when using styluses of different weights. From this he concluded that the limiting factor can not be the physical strength.

In analogy to Shannon's information theory Fitts assigned an index of difficulty $ID = \log_2(2A/W)$ to each of his motor task where A is the distance to the target (from Amplitude) and W the width of the target.

A Simple Model

There is an easy way to derive Fitts' law from a simple model. The model, often called discrete-step-model, is standard knowledge in Fitts' law research, but is reinvented here, as it was not possible to find the first author who presented it. In this model the pointer approaches the target in steps. In each step the pointer aims to the target center and reaches a position within an error circle from where the next step starts. Each step consumes the same amount of time (as it processes the same amount of bits) and brings the pointer gradually closer to the target. The process ends when the pointer is inside the target. See figure 1 for an illustration.

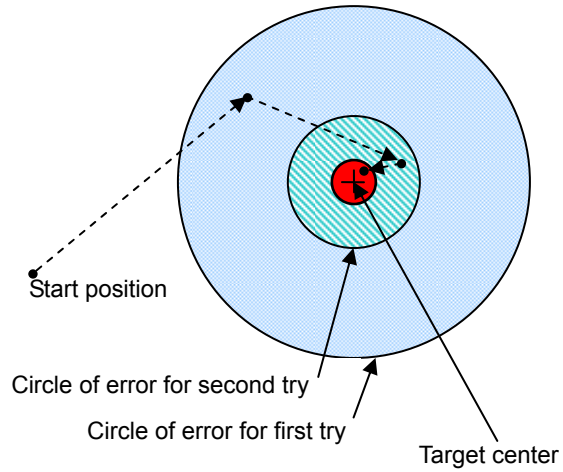

figure 1. Step-wise movement towards target

Let the distance to the target at each stage be A_i with the initial distance $A_0 = A$. After each step, the average distance to the center of the target A_{i+1} is a constant fraction λ of the distance A_i at the beginning of the step.

$$A_{i+1} = \lambda \cdot A_i$$

and consequently:

$$A_i = \lambda^i \cdot A$$

The process stops after n steps when the distance to the target center is less than the radius R of the target:

$$A_n = \lambda^n \cdot A < R$$

From this derives:

$$n = \log(R / A) / \log(\lambda)$$

Each step takes a fixed time τ and there will be some initial time *a* for the brain to get started. The total time *t* to reach the target is:

$$t = a + \tau \cdot n = a + \tau \cdot \log (R / A) / \log (\lambda)$$

$$t = a + b \cdot \log (A / R)$$

where $b = - \tau / \log (\lambda)$. As the pointer gets closer to the target with each step, λ is smaller than 1 and $\log(\lambda)$ is negative so *b* is positive.

The error circles represent a probability density for the inaccuracies of movement. The probability density can be symmetrical but this is not demanded. The derivation only demands that the expected value for the distance to the target after a step is proportional to the distance at the beginning of the step. This can also be the case for unsymmetrical probability densities for example when grabbing a cup where overshooting is not allowed.

It is Only a Model

The model is very simple and has its problems. For example, the speed of the pointer has its maximum at the initial move. However, the pointer is in rest at the beginning and physical object can not accelerate within zero time.

The model also neglects many factors which influence pointing actions of a subject. A real movement shows inaccuracies by tremor which is not related to the target distance. The model does not cover the subjects' mood, their nervousness, their eye-sight, chemical substances like alcohol or caffeine in their blood, or distraction by the environment.

However, the model has clear assumptions and allows a clear interpretation of the *a*- and *b*-constant (see the discussion below).

Variations of Fitts' Formula

Fitts did not give a Fitts' law formula in his publication. He only introduced the index of difficulty ID. Subsequent researchers suggested different variations for a formula of Fitts' law.

Frequent variations are:

$$t = a + b \cdot \log_2(2 \cdot A / W) \qquad (1)$$

$$t = a + b \cdot \log_2(A / W + 0.5) \qquad (2)$$

$$t = a + b \cdot \log_2(A / W + 1) \qquad (3)$$

$$t = a + b \cdot \log_2(A / W) \qquad (4)$$

In these formulas *t* is the mean time to hit the target. The constants *a* and *b* are measured in a Fitts' law experiment. Constant *a* is called the non-informational part and has the unit seconds. Constant *b* is the informational part in seconds per bit.

Formula 1 uses an ID as defined by Fitts. Formula 2 was introduced by Welford [8] and formula 3 by MacKenzie [5]. Factoring out the 2 in formula 1 is legal and leads to formula 4, however with a different value and interpretation for the *a*-constant.

It is worth to mention that the definition of the distance to the target influences the formula. Let A_c be the distance to the center of the target and A_e the distance to the edge of the target. The different distance definitions relate to each other by:

$$A_c = A_e + W/2$$

Consequently, the index of difficulty ID turns into:

$$\log_2(2 \cdot A_c / W) = \log_2(2 \cdot A_e / W + 1)$$

For this reason reports on a Fitts' law experiment should clearly state which definition of distance it uses. Sadly, many publications miss a clear statement.

Interpretation of Fitts' Law Constants

The *b*-constant is called informational part and has the unit seconds per bit. It states how much time is needed to process a bit. The *a*-constant has the unit seconds. For formula 1 it is the reaction time. The derivation explicitly added this constant as reaction time. In Formula 4 the interpretation of the *a*-constant has a different meaning:

$$t = a + b \cdot \log_2(2 \cdot A/W)$$

$$= a + b \cdot (\log_2(A/W) + \log_2(2))$$

$$= a' + b \cdot \log_2(A/W) \text{ with } a' = a + b$$

The calculation shows that the *b*-constant does not change by factoring out the 2. As the value of the a-constant changes according to the used formula, many authors call it non-informational part.

By the way, the ISO 9241-9 uses only one parameter, the throughput TP, to characterize pointing devices. It is questionable whether this single value adequately describes a pointing device (see Zhai's publication [9] for a detailed discussion). It is also questionable whether the ISO standard reflects the performance of a pointing device; the Fitts' law constants reflect the performance of the nervous system of humans.

The Confusion with Fitts' Formula

It was Fitts himself who started the confusion with the variations of his law in the way he introduced the factor 2:

"*The use of 2A rather than A is indicated by both logical and practical considerations. Its use insures that the index will be greater than zero for all practical situations and has the effect of adding one bit ($-\log_2 1/2$) per response to the difficulty index. The use of 2A makes the index correspond rationally to the number of successive fractionations required to specify the tolerance range out of a total range extending from the point of initiation of a movement to a point equidistant on the opposite side of the target.*" [2]

Actually, when just looking at the formula, the factor 2 does not ensure that the ID will be positive; adding 1 can achieve this. Nevertheless, Fitts is absolutely right. Perhaps Fitts did not express it in an elegant way, but within the analogy to information theory, he mapped the amplitude of the noise to the width of the target. However, the width of the target corresponds with the difference from peak to peak. In consequence, he took also the peek-to-peek value for the amplitude of the movement, which is 2A. Comparing Fitts' definition of the index of difficulty with the derived formula, it turns

out that both definitions are equal 2A/W = A/R. With this understanding the problem with negative IDs dissolves and Fitts is right with his statement "*the index will be greater than zero for all practical situations"*. For a negative ID the distance to the target has to be smaller than the radius. This means that the pointer is already inside the target and the entry to the target happened in the past. Nothing is wrong with formula 1.

The confusion within the HCI community started 1991 with the publication of MacKenzie, Sellen, and Buxton [6]. Their paper mentions three different formulas. The numbers for the formulas in the following citation are equal to the numbers used in this document. They wrote:

"*There is recent evidence that the following formulation is more theoretically sound and yields a better fit with empirical data (MacKenzie, 1989):*

$$MT=a+b \log2(A/W+ l). \quad (3)$$

In an analysis of data from Fitts' (1954) experiments, Equation 3 was shown to yield higher correlations than those obtained using the Fitts or Welford formulation. Another benefit of Equation 3 is that the index of difficulty cannot be negative, unlike the log term in Equation 1 or 2. Studies by Card et al. (1978), Gillan, Holden, Adam, Rudisill, and Magee (1990), and Ware and Mikaelian (1987), for example, yielded a negative index of difficulty under some conditions. Typically this results when wide, short targets (viz., words) are approached from above or below at close range. Under such conditions, A is small, W is large, and the index of difficulty, computed using Equation 1 or 2, is often negative. A negative index is theoretically unsound and diminishes some of the potential benefits of the model." [6]

From the discussion above it is clear that negative IDs can not happen with formula 1. A negative ID for formula 1 means that the pointer is already inside the target. Otherwise it means that target size or distance are not proper defined and not that there is something wrong with the formula.

In addition, there are doubts whether formula 3 is more theoretically sound. MacKenzie writes:

"*Shannon's Theorem 17 expresses the effective information capacity C (in bits × s-1) of a communications channel of band B (in s-1) as*

$$C = B \log2((P + N) / N) \quad (4)$$

where P is the signal power and N is the noise power (Shannon & Weaver, 1949, pp. 100-103).

It is the purpose of this note to suggest that Fitts' model contains an unnecessary deviation from Shannon's Theorem 17 and that a model based on an exact adaptation provides a better fit with empirical data. The variation of Fitts' law suggested by direct analogy with Shannon's Theorem 17 is

$$MT = a + b \log2((A + W) / W)$$

It is revealing to examine the source Fitts cites in his paper at the point where he introduces the relationship (Fitts, 1954, p. 368). His derivation is based on Goldman's Equation 39 (Goldman, 1953), which is

similar to Fitts' law except in its use of the terminology of communications systems:

$$C = B \log_2(P / N) \qquad (6)$$ " [5].

MacKenzie states that his direct analogy provides a better fit with empirical data, but he does not give a theoretical explanation why Fitts' model contains an unnecessary deviation. He also does not explain what is analogue to what and why it is legitimate to apply an analogy.

The simple questions regarding the direct analogy are: Why does the power of the noise map to the target width, which means the diameter, and not to the radius? Amplitudes should map to radius or half of the target width respectively (see the introduction of factor 2 by Fitts). The power of the noise is proportional to the square of the noise amplitude (or variance of the noise, see also the footnote in Fitts' publication [2]). Therefore, a further question is what happened to the square? In the case of Goldman's Equation the square can be drawn out of the logarithm and doubles *B*. Is it possible that the direct analogy – take the distance as the signal power and the diameter as the power of the noise – is a little bit too direct or in other words naïve?

MacKenzie continues with:

"*Fitts recognized that his analogy was imperfect. The "2" was added (see Equation 1) to avoid a negative ID when A = W ; however, log2(2A / W) is zero when A = (W / 2) and negative when A < (W / 2). These conditions could never occur in the experiments Fitts devised. Other researchers, however, have reported experimental conditions with ID less than 1 bit (Drury, 1975), or with a negative ID (Crossman & Goodeve, 1983; Ware and Mikaelin, 1987). It is noteworthy that, in the model based on Shannon's theorem (see Equation 5), ID cannot be negative.*" [5]

Actually, Fitts did not state that his analogy is imperfect. Again and as already discussed above, the ID can not become negative in Fitts' formula (as long as the pointer is outside the target). It is also not clear what level of understanding the citation addresses. People who are familiar with information theory do not need an explanation under which conditions log(2A/W) is negative. Doubts that MacKenzie's formulation is more theoretically sound seem to be justified.

MacKenzie claims that empirical data provides a better correlation with his formula. The question whether a better correlation proves a formula is left to the statistics experts. Definitely, it does not provide new insights or deeper understanding.

Evaluation of Fitts' Law Data

It seems that it is common in the HCI community to give a correlation for the data recorded in a Fitts' law experiment and to argue that the data fit well if the correlation is close to 1. One example for arguing with correlation was given already in the introduction. Another example is the application of Fitts' law to eye movements done by Miniotas [7] where he presents extremely good correlations and concludes that therefore Fitts' law applies to eye movements. This publication is also a good example for blind application of Fitts' law to any pointing action. There is no reason to assume Fitts' law for eye pointing. There is only a small area (≈ 1°) of high resolution on the retina (fovea) and therefore the eye sees only a small spot

clearly. The concept of target size involves object recognition and typically, the saccadic movement of the eye takes place for this purpose. All textbooks in psychology state that the eye performs ballistic movements and obey the formula of Carpenter (1977) which has no dependency on target sizes. Building IDs without showing a target size dependency as done by Miniotas, however, forces Fitts' law to be valid. An evaluation of Miniotas' experiment based on Carpenter's formula produces also a good correlation.

However, a correlation not equal to zero is only a statistical indicator for a linear dependency of two variables. The question whether this indication has significance needs a further test. The result of this test depends on the number of data pairs recorded in the experiment. If there are more data pairs for the evaluation, the correlation can be lower for the same level of significance. Therefore, the strategy mentioned in the introduction to use only a small number of IDs to gain a good correlation does not make sense. The common practice to build an average on execution times for one ID first and calculate the correlation afterwards with a small number of IDs provides good correlation values, but does not prove anything.

It is easy to demonstrate the effect with data recorded from an experiment (done by the author). Figure 2 plots the performance of one participant for a target acquisition task with a classical mouse against the index of difficulty. The measurement was done with two different target sizes; 50 clicks for each target size and randomly chosen distances. The b-constant of Fitts' law, which is the slope of the trend line, calculates to 82 ms/bit and R^2 to 0.17, which is very low.

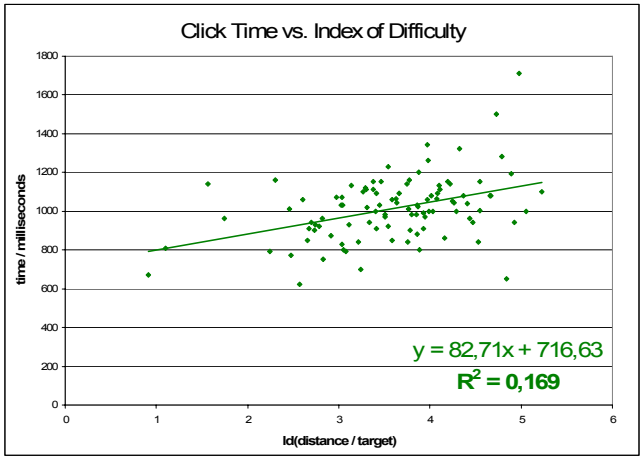

figure 2. Performance of one participant for a click-the-target task with a classical mouse

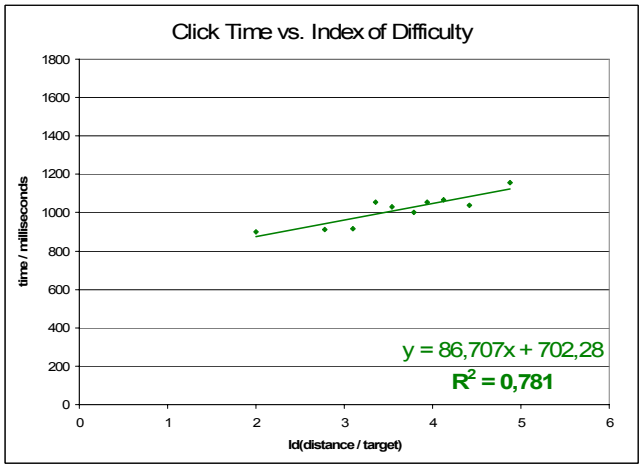

figure 3. The same data from figure 2, but averaged. One point represents an average of ten clicks.

Figure 3 plots the same data, but after sorting and averaging the data by groups of ten. Then the value for the b-constant is 86 ms/bit and nearly unchanged. However, the squared correlation coefficient R^2 now is 0.78. With fewer data points the R^2 value is much better. Consequently, the correlation alone does not tell how well the data fit here.

For practical purposes it would be more useful to state the accuracy of the measured constants. A statement like "Our b-constant has the value 0.12 ± 0.03 s/bit" would be more helpful than the statement "Our data show a correlation r = 0.9910".

Discussion

Despite decades of research and hundreds of publications on Fitts' law it seems that HCI community still needs clarifications on the topic. It also seems that Fitts' law is overused and its importance for HCI is highly overrated. With respect to the extreme noise, which is typical for Fitts' law experiments, the exact formula is not very important. In contrast, the range of the noise is important for HCI as it defines the expected range of input for a user interface.

Fitts' law tells that it takes more time to hit a target if the target is smaller or further away. This is a statement in accordance with common sense and sufficient for most purposes of HCI. Space for buttons on a chassis or a display is a limited resource. For this reason the button size should not be too big. On the other hand the buttons should not be too small as this increases the time to hit the button and time is also a limited resource. The task of HCI is to find a good balance and this can already be achieved with common sense.

For the future it would be nice if the HCI community could agree on a standard model for Fitts' law and a corresponding formula. There is no reason not to take Fitts' formula for Fitts' law.

Of course, every model has its limits and does not explain reality completely. However, it is not always helpful to emphasize that in reality the things are much more complicated. The benefit of a model lies in its simplification. The models in physics, for example, allow giving a formula for the motion of a perfect sphere on a perfect surface, but not of a stone thrown down a hill.

An agreement on a standard model would bring several benefits for researchers, teachers, and students. A researcher could refer to the standard model without a justification for the choice of the formula used for the evaluation. At the moment, referring to Fitts' law provokes a long discussion and shifts the focus away from the original research question. A teacher could explain Fitts' law without the problematic question which formula is correct. For the student who honestly seeks the scientific truth the current situation is a disillusion. A standard model could give back the student's confidence into scientific literature.

Conclusion

For two decades the HCI community lives with different formulas for Fitts' law. It is obvious that the different formulas contradict each other. Instead of a clarification which formula is correct the HCI community gives free choice but demands good statistical values for the chosen formula. This is not a scientific approach as science does not allow contradictions and inconsistencies.

Normally, when there are different formulas for the same thing, the formulas were derived from different assumptions and an experiment serves to decide which assumption is wrong. However, in the case of Fitts' law the different formulas are derived from the same assumption, meaning a vague analogy to Shannon's information theory. Consequently, there must be something wrong with some derivations.

It is hard to imagine that within twenty years nobody noticed this inconsistency. Therefore, it seems to be more likely that it is difficult in the HCI community to publish critical content. Typically, the people whose scientific work is criticized are among the reviewers. The double-blind review process does not reveal the identity of the reviewers and they never have to take responsibility for the review they wrote. The person whose justified critics were not heard will most probably leave the community.

The long period of confusion on Fitts' law caused severe damage. The references to the different formulas spread over the scientific publications of the last twenty years. It is not possible to correct this without declaring big parts of the HCI literature as obsolete.

There has been massive critics on scientific standards in the HCI community by Greenberg and Buxton [3] recently. Problems mentioned are missing replication of user studies and inadequate application of scientific methods. The situation with Fitts' law presented here confirms that there are severe problems.

The question is what the consequences are. One possibility would be to make HCI an art. In this case the community could live with contradictions and more than one truth. However, if HCI wants to be science, the community should discuss how to ensure scientific standards.

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